Effects of dephasing on shot-noise in an electronic Mach-Zehnder interferometer

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We present a theoretical study of the influence of dephasing on shot noise in an electronic Mach-Zehnder interferometer. In contrast to phenomenological approaches, we employ a microscopic model where dephasing is induced by the fluctuations of a classical potential. This enables us to treat the influence of the environment's fluctuation spectrum on the shot noise. We compare against the results obtained from a simple classical model of incoherent transport, as well as those derived from the phenomenological dephasing terminal approach, arguing that the latter runs into a problem when applied to shot noise calculations for interferometer geometries. From our model, we find two different limiting regimes: If the fluctuations are slow as compared to the time-scales set by voltage and temperature, the usual partition noise expression $\mathcal{T}(1-\mathcal{T})$ is averaged over the fluctuating phase difference. For the case of "fast" fluctuations, it is replaced by a more complicated expression involving an average over transmission amplitudes. The full current noise also contains other contributions, and we provide a general formula, as well as explicit expressions and plots for specific examples.

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I. INTRODUCTION

A large part of mesoscopic physics is concerned with exploiting and analyzing quantum interference effects in micrometer-size electronic circuits. Therefore, it is important to understand how these interference effects are diminished by the action of a fluctuating environment (such as phonons or other electrons), both in order to estimate the possibilities for applications of these effects as well as for learning more about the environment itself. This holds for "bulk" effects such as universal conductance fluctuations and weak localization, but also for interference in microfabricated electronic interference setups, such as various versions of "double-slit" or "Mach-Zehnder" interferometers, often employing the Aharonov-Bohm phase due to a magnetic flux penetrating the interior of the interferometer. In recent years, many studies[1,2,3,4,5,6,7] have been performed to learn more about the mechanisms of dephasing and the dependence of the dephasing rate on parameters such as temperature. One very delicate issue in the analysis of these experiments is the fact that the "visibility" of the interference pattern can also be diminished by thermal averaging, when electrons with a spread of wavelengths (determined by voltage or temperature) contribute to the current. Recently, an ideal single-channel electronic Mach-Zehnder interferometer has been realized experimentally for the first time[8]. The arms of the interferometer have been implemented as edge channels of a two-dimensional electron gas in the integer quantum hall effect regime. Besides measuring the current as a function of voltage, temperature and phase difference between the paths, the authors also measured the shot noise to be able to distinguish between mere "phase averaging" and genuine dephasing. Although the interpretation of the experimental results still remains unclear to some extent, the idea of using shot noise to learn more about dephasing is a very promising one, as it connects two fundamental topics in mesoscopic physics.

Most theoretical works on dephasing in mesoscopic interference setups are concerned with its influence on the average current only (see Refs. 9,10,11,12,13,14 and references therein). Nevertheless, in some works[15], the effects of dephasing on shot noise have been studied, employing the phenomenological "dephasing terminal" approach[15,16,17,18,19], where an additional artificial electron reservoir randomizes the phase of electrons going through the setup. However, this approach does not include any information about the power spectrum of the fluctuations in the environment, which, from other studies, is known to play an important role in discussions of dephasing. Therefore, in the present work, we have set ourselves the task to analyze the effects of dephasing on the shot noise in a model that incorporates the fluctuation spectrum.

Apart from that, the model is deliberately chosen as simple as possible: The case of a true "quantum bath" will not be treated here. It is the case relevant for lower temperatures and higher voltages, when dephasing is produced primarily by spontaneous emission of energy into the bath. This case immediately leads to a many-body problem where the Pauli principle plays an important role. Instead, we will consider a Mach-Zehnder setup (Fig. 1) where the electrons are subject to the fluctuations of a classical noise field[11] (transmission phases become a Gaussian random process in time). This describes the case of noisy nonequilibrium radiation impinging on the system, as well as the effects of the classical part

of the noise spectrum of a quantum-mechanical environment, which should dominate at higher temperatures. The advantage of a classical noise field is that we still can use a single-particle picture. We will also compare and contrast our results with those obtained either from a very simple classical model of dephasing or the dephasing terminal. Regarding the dephasing terminal, we will argue that its application to shot noise in interferometer geometries is most likely plagued with a certain problem that artificially changes the shot noise result. To the best of our knowledge, the work presented here is the first microscopic analysis dealing with the effects of dephasing on shot noise in any electronic two-way interferometer geometry. This study provides the basis for dealing with the influence of time-varying potentials on the shot noise in other two-way interferometer geometries, as well as for extensions to the case of a true quantum-mechanical environment.

We will demonstrate that the results depend strongly on whether the environmental fluctuations are fast or slow with respect to the timescales set by voltage and temperature. In the "slow" case, the shot noise merely becomes equal to the phase-average of the usual partition noise expression[20], while the expression for the other limit is more complicated than that. A brief discussion of a part of the results has already been presented elsewhere[21]. Recently, an analysis of dephasing in a mesoscopic resonant-level detector has been carried out along similar lines, including the effects on shot noise[22].

Our work is organized as follows: After discussing the reduction in visibility of the current interference pattern (Sec. II), we explain the basic idea behind using shot noise as a tool to distinguish genuine dephasing from mere phase averaging (Sec. III). The influence of dephasing on shot noise is then derived both for a simple classical model (Sec. IV) and from the dephasing terminal approach (Sec. V). Both of these models are phenomenological, and we explain why we believe there is a problem with the dephasing terminal approach, as applied to shot noise calculations in interferometer geometries (Sec. VI). Then we turn to the model of dephasing by a classical fluctuating potential (Sec. VII), which permits to take into account the power spectrum of the environment, in contrast to the other approaches. We will discuss the general current noise formula (Eqs. (40),(41)), as well as limiting cases (Sec. VII D) and plots for special examples (Sec. VII F). Finally, we will compare the results of the various different models and regimes (Sec. VIII).

II. VISIBILITY

The transmission probability (and thus the current) is determined by squaring the sum of transmission amplitudes related to the two arms of the interferometer. This results in a sum of two classical probabilities, plus an interference term depending on both amplitudes:

$$I \propto |A_L|^2 + |A_R|^2 + A_L^* A_R \left\langle e^{i\delta\varphi} \right\rangle_\varphi + c.c. \qquad (1)$$

The term $A_L^* A_R$ may contain a fixed Aharonov-Bohm phase factor, as well as a phase factor $\exp(ik\delta x)$ related to a possible path-length difference between the two arms. In addition, there may be an extra fluctuating phase difference $\delta\varphi$, due to the action of a fluctuating environment, and we have displayed it explicitly in Eq. (1). In evaluating the current, one has to perform the average of this fluctuating phase factor $\exp(i\delta\varphi)$, which will result in a number of magnitude less than one. This decreases the interference term and therefore the visibility of the "interference pattern", which is represented by the dependence of $I(\phi)$ on the controllable Aharonov-Bohm phase difference ϕ between the paths. The visibility is commonly defined as

$$(I_{max} - I_{min})/(I_{max} + I_{min})\,, \qquad (2)$$

where the maximum and minimum of the current is calculated with respect to ϕ. This takes on the optimal value of 1, if the interference term is not suppressed and the two beam splitters are symmetric ($|A_L| = |A_R|$). Any fluctuations in $\delta\varphi$ decrease the visibility. However, besides temporal fluctuations in $\delta\varphi$, there is another, more trivial, effect that can diminish the visibility, if there is a finite path-length difference δx: This gives rise to an additional factor $\exp(ik\delta x)$ in the interference term, which has to be averaged over a range of k-values determined by voltage and temperature (we will call this "thermal averaging", for brevity). Therefore, in that case the visibility also decreases upon increasing voltage or temperature.

Therefore, on the level of the average current $I(\phi)$, dephasing cannot be distinguished easily from thermal averaging. Generically, even the qualitative dependence of these two different effects on the parameters (V, T etc.) will be similar, since increasing temperature and/or the bias voltage will usually also increase dephasing. Nevertheless, in the present model a striking difference appears if the dependence on the bias voltage is analyzed in more detail: The average of $\exp(ik\delta x)$ does not simply decrease with increasing bias voltage, but shows an oscillatory behaviour. Let us illustrate this briefly in the special case of $T = 0$, when the electrons contributing to the current are injected from the input reservoir in a voltage window corresponding to a box-distribution of k-values ($k \in [\bar{k} - \delta k/2, \bar{k} + \delta k/2]$). Then we get

$$\langle \exp(ik\delta x) \rangle_k = e^{i\bar{k}\delta x} \frac{\sin(\delta k \delta x/2)}{\delta k \delta x/2}\,, \qquad (3)$$

which is an oscillatory function that will lead to zero visibility at all bias voltages for which $\delta k \delta x$ is an integer multiple of 2π. Finte temperatures will diminish the average further but will not destroy the oscillatory

behaviour. Such an effect, if present, should be easily confirmed in an experiment.

However, no such observation has been reported in the Mach-Zehnder experiment[8]. In general, this absence of oscillations in the visibility as a function of bias voltage could be taken as a strong hint for the importance of genuine dephasing, provided our idealized model applies. In addition, note that there would be a voltage-dependent phase-shift $\bar{k}\delta x$ of the interference pattern, via Eq. (3), which could be used to derive the value of δx (provided the Fermi velocity is known), and which could be checked against the period of the oscillation in the visibility, which is determined by δx as well. Furthermore, it should be noted that the voltage-dependence of the visibility plotted in Ref. 8 was not obtained by simply measuring at different bias voltages. Instead, a dc-voltage was increased while measuring the ac-current flowing due to a small ac-modulated voltage on top of the dc bias. Ideally, the visibility of the ac signal should not decrease with dc-voltage, if the supression of the interference term were not affected by dephasing but only by thermal averaging. The actual observation of a decrease in visibility could therefore be interpreted as another sign ruling out thermal averaging. Unfortunately, one cannot be sure that the change of bias voltage does not affect the transmission amplitudes themselves[23], and this in turn could mean that the ac visibility is affected by electron transmission in a wider range of wavelengths. Thus no firm conclusions can be drawn from the reported measurements of the voltage dependent visibility.

III. SHOT NOISE AS A MEASURE OF DEPHASING: BASIC IDEA

Apart from a quantitative analysis of the temperature- and voltage-dependence of the interference visibility, there exists another, potentially more powerful way to distinguish simple thermal averaging from dephasing: shot noise. This was already pointed out in Ref. 8. The basic idea is that the partition noise $\propto \mathcal{T}(1-\mathcal{T})$ is nonlinear in the transmission probability $\mathcal{T}$, such that it matters whether averaging is performed before or after calculating this expression. Thermal averaging of the independent shot-noise contributions from different k amounts to an expression of the form

$$\langle \mathcal{T}(1-\mathcal{T})\rangle_k \, . \tag{4}$$

An analogous expression is expected to hold if some parameter fluctuates slowly from run to run of the experiment, such that the partition noise should be averaged over this parameter. In contrast, for the purposes of interpreting the measurements[8], dephasing was assumed to lead to partition noise of the form

$$\langle \mathcal{T}\rangle_\varphi \left(1-\langle \mathcal{T}\rangle_\varphi\right), \tag{5}$$

where $\langle \mathcal{T}\rangle_\varphi$ denotes the transmission probability whose interference term is already suppressed (partially) due to dephasing.

Even if the visibility turns out to be the same in both cases, the shot noise expressions (4) and (5) are quite different: For example, in the special case of zero visibility and 50% transmission of the first beamsplitter ($T_A = 1/2$), the shot noise depends on the transmission of the second beam splitter, T_B, only in the case of thermal averaging[8], Eq. (4). In the case of dephasing, the shot noise in Eq. (5) turns out to be independent of T_B. In the next section, we will show that Eq. (5) indeed follows from generalizing the result of a phenomenological classical model for shot noise of incoherent electrons.

On the other hand, it is clear that the ansatz (5) cannot possibly hold in all parameter regimes: In particular, if the environment-induced fluctuations of the phase are sufficiently slow (compared with the frequency scale set by temperature or bias voltage, see below), we would expect that their effect will be just the same as that of thermal averaging (or that of parameters fluctuating from run to run of the experiment), leading to a formula similar to Eq. (4). A qualitative picture is the following: We may view the current as being composed of a stream of wave packets entering the interferometer, each of them of a temporal width equal to the correlation length, i.e. $\min(1/k_BT, 1/eV)$. After the final beam splitter, the probability weights of the two parts of the wave packet are determined by the phase difference between the two interfering paths L and R. If the fluctuations of this phase happen on times much shorter than the temporal extent of the packet, the probability of detecting the particle in either output port will be 50/50 , for *each* packet that enters the interferometer (in a symmetric setup, with large fluctuations of the phase, leading to zero visibility). This situation is depicted in Fig. 5. On the other hand, if the fluctuations are slow compared to this time scale, then each packet sent through the setup will feel a fixed (but random) phase, such that the effects (also in terms of shot noise) are indistinguishable from thermal averaging. This will be confirmed by the microscopic model of section VII.

IV. PHENOMENOLOGICAL CLASSICAL MODEL

We start from a classical model for shot-noise in a completely incoherent electronic Mach-Zehnder interferometer setup (see Fig. 1).

For simplicity, we consider a Mach-Zehnder setup at $T = 0$, with a voltage V applied between the source 1 and the other terminals. A heuristic model[24] for shot noise calculations consists in assuming the source emitting electrons in regular intervals of frequency eV/h. It is well-known that this model yields the correct quantum-mechanical result for the partition noise of a single barrier, when the variance of the number of transmission

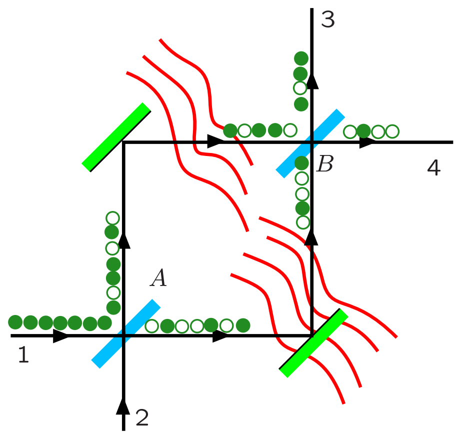

Figure 1: A simple classical model for the fully incoherent case: Electrons impinging regularly onto the Mach-Zehnder interferometer and making classical stochastic choices at both beam splitters.

events is calculated. We now go further and implement full loss of phase coherence inside the interferometer by using classical probability theory to describe the stochastic choices the electron makes at *each* of two beam splitters (instead of squaring complex amplitudes describing the coherent passage through the full device).

Within this model, we can consider the current at the ouput terminal, I_3, to be a dichotomous random number (0 or 1), whose value depends on whether the given electron reaches the output port 3 (nothing essential changes for port 4, other than interchanging transmission and reflection amplitudes at the second beam splitter). We obtain

$$\langle I_3 \rangle = T_A T_B + R_A R_B \equiv \langle \mathcal{T} \rangle_\varphi , \; \langle \delta I_3^2 \rangle = \langle \mathcal{T} \rangle_\varphi (1 - \langle \mathcal{T} \rangle_\varphi) , \tag{6}$$

where $\delta I_3 = I_3 - \langle I_3 \rangle$ and we have denoted the fully incoherent transmission probability by $\langle \mathcal{T} \rangle_\varphi$. The shot noise expression derived from this simple model therefore agrees (in the fully incoherent limit) with the ansatz considered in Eq. (5). Unfortunately, the generalization to arbitrary (partial) coherence cannot be made within the present model. Therefore, we turn to a more sophisticated but still phenomenological approach which works for any value of the visibility.

V. DEPHASING TERMINAL APPROACH

In this section, we analyze shot noise for a one-channel Mach-Zehnder setup, employing the dephasing terminal approach[15,16,17,18,19]. This will enable us to treat the case of arbitrary visibility, although it is still not possible to incorporate the spectrum of environmental fluctuations (see Section VII). As we will see, the dephasing terminal

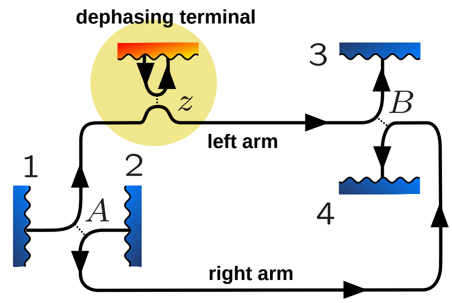

Figure 2: The Mach-Zehnder interferometer setup considered in the text: At beam splitters A and B the electrons are transmitted with amplitudes $t_{A,B}$. The fictitious reservoir φ serves as a "dephasing terminal". The coherence parameter z denotes the amplitude for an electron to be reflected at the beam-splitter connecting the left arm of the interferometer to the reservoir φ (thus $z = 1$ for fully coherent transport).

model leads to a shot noise expression which, in general, differs both from $\langle \mathcal{T}(1-\mathcal{T}) \rangle$ and $\langle \mathcal{T} \rangle (1 - \langle \mathcal{T} \rangle)$.

Our aim is to calculate the noise of the output current at terminal 3 of the interferometer shown in Fig. 2. The basic idea behind the dephasing terminal is to mimick the effects of dephasing on transport in a mesoscopic conductor by attaching a fictitious extra reservoir to the setup[15,16,17,18,19]. In order to correctly describe pure dephasing, it is essential to force the current into this dephasing terminal to vanish at each energy and instant of time. Both the average electron distribution in the terminal as well as its fluctuations have to be chosen appropriately to fulfill this condition.

We assume the dephasing terminal φ to be attached to the left interferometer arm (without loss of generality, see below). The arms are treated as chiral edge channels (see Fig. 2). The amplitude for an electron to move on coherently, without entering the reservoir, is assumed to be $z \in [0, 1]$. When an electron enters the reservoir with probability $1 - z^2$, it "loses its phase" and is re-emitted afterwards. In this way, z describes the coherence, with $z = 1$ corresponding to fully coherent transport and $z = 0$ to the completely incoherent case. The amplitude for an electron to go from reservoir β into reservoir α is denoted by the scattering matrix amplitude $s_{\alpha\beta}$. Assuming backscattering to be absent at the beamsplitters, the setup of Fig. 2 yields the following S-matrix amplitudes:

$$\begin{aligned} s_{3\varphi} &= i r_B e^{i\phi} \sqrt{1 - z^2} \quad , \\ s_{31} &= t_A t_B + z r_A r_B e^{i\phi} \quad , \quad s_{\varphi 1} = i r_A \sqrt{1 - z^2} \\ s_{32} &= r_A t_B + z t_A r_B e^{i\phi} \quad , \quad s_{\varphi 2} = i t_A \sqrt{1 - z^2} \end{aligned} \tag{7}$$

and $s_{\varphi\varphi} = z$, $s_{33} = s_{\varphi 3} = s_{2\varphi} = s_{1\varphi} = 0$. Here t_A, r_A and t_B, r_B are transmission and reflection amplitudes at the beam splitters A and B (with $|r_j|^2 + |t_j|^2 = 1$ and $r_j^* t_j = -r_j t_j^*$). The total phase difference ϕ between the two paths is assumed to include both a pos-

sible Aharanov-Bohm phase, ϕ_{AB}, as well as the effect of unequal path lengths, $k\delta x$ (which makes ϕ energy-dependent). Note that, within this model, there is no extra "fluctuating phase difference" $\delta\varphi$, since dephasing is already included phenomenologically by the presence of the dephasing terminal.

The current flowing *out* of reservoir α at energy E and time t is given by (note $\hbar \equiv 1$)

$$I_\alpha(E,t) = \frac{e}{2\pi}(f_\alpha - \sum_\beta |s_{\alpha\beta}|^2 f_\beta) + \delta I_\alpha \,, \qquad (8)$$

where δI_α denotes the original current fluctuations (at E, t) calculated in the absence of any additional fluctuations of the distribution functions f_α (see below).

Following the calculation of Ref. 19, we demand the current flowing into the dephasing terminal φ to be zero at each energy and point in time, including its fluctuations. By solving the equation $I_\varphi(E,t) \equiv 0$ for f_φ, we obtain:

$$f_\varphi = \left[-\frac{2\pi}{e}\delta I_\varphi + \sum_{\beta\neq\varphi} |s_{\varphi\beta}|^2 f_\beta \right] \left[1 - |s_{\varphi\varphi}|^2\right]^{-1} . \qquad (9)$$

The current fluctuations δI_φ on the right hand side determine the required fluctuations δf_φ of $f_\varphi(E,t) = \delta f_\varphi(E,t) + \bar{f}_\varphi(E)$.

Inserting $\bar{f}_\varphi = R_A f_1 + T_A f_2$ with $T_A = |t_A|^2$, $R_A = 1 - T_A$ into the averaged Eq. (8), we obtain the energy-integrated average current at the output port $\alpha = 3$:

$$\bar{I}_3 = \frac{e}{2\pi}\int dE\, \left[f_3 - f_1 \langle T_1\rangle - f_2 \langle T_2\rangle\right] \,, \qquad (10)$$

where the probabilities of transmission from terminals 1 and 2 to terminal 3 are denoted by $\langle T_1\rangle$ and $\langle T_2\rangle$. The notation $\langle T_j\rangle$ is chosen to signal that these transmission probabilities are already affected by dephasing: They contain an interference term which is multiplied by the amplitude z of coherent transmission:

$$\langle T_1\rangle = T_A T_B + R_A R_B + 2z(t_A^* r_A)(t_B^* r_B)\cos\phi \,, \qquad (11)$$

and $\langle T_2\rangle = 1 - \langle T_1\rangle$. For the purposes of calculating the current, the effect of dephasing may be thought of as an average of the fully coherent expression ($z = 1$) over a fluctuating extra contribution $\delta\varphi$ to the phase difference ϕ. This average leads to the suppression of the interference term: $\langle\cos(\phi + \delta\varphi)\rangle = z\cos\phi$. Thus, no simple distinction between genuine dephasing and phase averaging is possible at this level. The energy-integration in (10) may result in an additional suppression, if there is a difference in the path lengths of the two interferometer arms (such that ϕ is energy-dependent).

As the phase difference between the two arms is varied (through a magnetic flux), the current $\bar{I}_3$ displays sinusoidal oscillations. The visibility of this interference pattern, $(I_{max} - I_{min})/(I_{max} + I_{min})$, is proportional to z. If energy averaging is not effective ($\delta k\delta x \ll 1$ with $\delta k = \max(k_B T, eV)/\hbar v_F$), the visibility is equal to $2z\sqrt{T_A R_A T_B R_B}/(T_A T_B + R_A R_B)$.

The *full* current fluctuations ΔI_α at $\alpha \neq \varphi$ contain both the usual fluctuations δI_α, as well as those induced by the additional fluctuations δf_φ of the distribution function in terminal φ:

$$\Delta I_\alpha = \delta I_\alpha - \frac{e}{2\pi}|s_{\alpha\varphi}|^2 \delta f_\varphi = \delta I_\alpha + \frac{|s_{\alpha\varphi}|^2 \delta I_\varphi}{1 - |s_{\varphi\varphi}|^2} \,. \qquad (12)$$

In particular, in our model we obtain

$$\Delta I_3 = \delta I_3 + R_B \delta I_\varphi \qquad (13)$$

for the full current fluctuations at the output port (terminal 3). In order to calculate the correlator of ΔI_3, we have to know the correlators of δI_3 and δI_φ (derived for $\delta f_\varphi = 0$). According to the scattering theory of shot noise[17,18,24,25], we have in general:

$$\begin{aligned} P_{\alpha\beta} \equiv 2\int dt\, \overline{\delta I_\alpha(t+t_0)\delta I_\beta(t_0)} = \\ 2\frac{e^2}{2\pi}\int dE \sum_{\gamma,\delta} f_\gamma(1 - f_\delta) \\ \times(\delta_{\alpha\gamma}\delta_{\alpha\delta} - s^*_{\alpha\gamma}s_{\alpha\delta})(\delta_{\beta\gamma}\delta_{\beta\delta} - s^*_{\beta\delta}s_{\beta\gamma}). \end{aligned} \qquad (14)$$

The overbar denotes a time-average over t_0, and the sums run over all terminals, including the dephasing terminal, where one has to put $f_\varphi = \bar{f}_\varphi$ for the purposes of this equation. Given these correlators, we can calculate the noise power at the output port of the interferometer:

$$\begin{aligned} 2S_{33} \equiv\ & 2\int dt\, \overline{\Delta I_3(t+t_0)\Delta I_3(t_0)} = \\ & P_{33} + 2R_B\, P_{3\varphi} + R_B^2 P_{\varphi\varphi}. \end{aligned} \qquad (15)$$

For simplicity, we first focus on the special case of zero temperature and no path-length difference (ϕ energy-independent). A bias voltage V is applied between terminal 1 and the other terminals: $f_1(E) = \theta(\epsilon_F + eV - E)$, $f_2(E) = f_3(E) = \theta(\epsilon_F - E)$. From (14), (15) and the scattering matrix amplitudes, we find :

$$\left(\frac{e^3 V}{2\pi}\right)^{-1} S_{33} = \langle T_1\rangle\langle T_2\rangle - 2(1 - z^2)R_A R_B T_A T_B \,. \quad (16)$$

Apparently, even for the fully incoherent case $z = 0$ the shot noise is *not* given by the simple expression $\langle T_1\rangle\langle T_2\rangle = \langle T_1\rangle(1 - \langle T_1\rangle)$, involving the product of averaged transmission probabilities (contrary to the result of

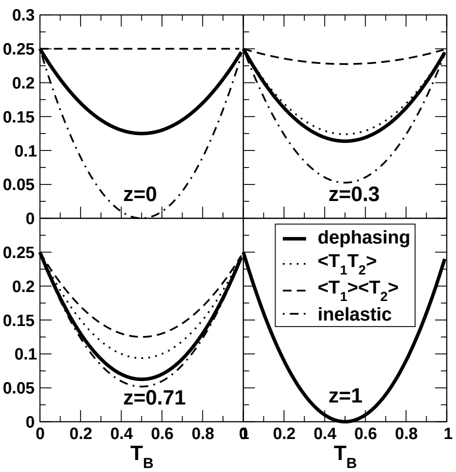

Figure 3: Normalized noise power $S_{33}/(e^3V/2\pi)$ vs. transmission probability T_B of second beamsplitter: Pure dephasing (Eq. (16), thick line) compared with the phase-averaged partition noise expression $\langle T_1T_2\rangle$ ($T_2 = 1 - T_1$), the product of phase-averaged probabilities, $\langle T_1\rangle \langle T_2\rangle$, and inelastic scattering (Eq. (18), symmetric case $\lambda = 1/2$). Different panels show various values of the coherence parameter z, with a maximum deviation between pure dephasing and the phase averaged result at $z = 1/\sqrt{2} \approx 0.7$. Other parameters: $T_A = 1/2$, $\phi = 0$.

the simple classical model in the previous section). However, it is interesting to note that this expression would indeed be found if one were to demand only the *average* current into the dephasing terminal to vanish at each energy, while we have also taken into account the restriction for the current fluctuations themselves. We will comment further on this difference between the two models in the next section. For the remainder of this section, we will just discuss the consequences of Eqs. (15) and (16).

We note that the result is independent of the location of the dephasing terminal: Indeed, placing the terminal into the right arm amounts to the replacements $\phi \mapsto -\phi$, $t_A \leftrightarrow r_A$, $t_B \leftrightarrow r_B$, which leave Eq. (16) invariant. More generally, repeating the analysis with a dephasing terminal in each arm gives exactly the same results as before, with $z = z_L z_R$ the product of the amplitudes for coherent transmission in each arm. Physically this is to be expected, since the effect of dephasing is only to scramble the *relative* phase between the two paths.

In order to compare expression (16) to the result of a phase-averaged partition noise expression, $\langle T_1T_2\rangle$, we have to evaluate $\left\langle\cos^2(\phi+\delta\phi)\right\rangle = (1 + \langle\cos(2(\phi+\delta\phi))\rangle)/2$. This is not simply related to z (which has been defined via the average of $\cos(\phi+\delta\phi)$). However, if we assume the phase fluctuations $\delta\phi$ to be Gaussian distributed, then $\langle\cos(2(\phi+\delta\phi)\rangle = z^4\cos(2\phi)$. In that case we obtain:

$$\langle T_1T_2\rangle - \langle T_1\rangle\langle T_2\rangle = \langle T_1\rangle^2 - \left\langle T_1^2\right\rangle = 4R_AR_BT_AT_B(\langle\cos\phi\rangle^2 - \left\langle\cos^2\phi\right\rangle) = -2R_AR_BT_AT_B(1-z^2)(1-z^2\cos(2\phi)) \quad (17)$$

We conclude that for zero visibility ($z = 0$) the shot noise expression (16) is equal to $\langle T_1T_2\rangle$, i.e. it has the form expected from a simple phase average! Therefore, according to the dephasing terminal approach, in this particular limit a shot noise measurement could not be used to distinguish phase averaging and genuine dephasing.

The coincidence between phase averaging and dephasing holds only at $z = 0$ (and, trivially, at $z = 1$). The difference between $\langle T_1T_2\rangle$ and the expression given in Eq. (16) is maximized if $T_A = T_B = 1/2$, $\phi = 0, \pi, 2\pi, \ldots$ and $z^2 = 1/2$. At these parameter values, the shot noise expression is 30% below the value of $\langle T_1T_2\rangle$, see Fig. 3.

If phase averaging (against which the pure dephasing case is to be compared) is actually due to energy integration over a phase factor $\exp(ik\delta x)$, then the distribution of $\delta\phi$ is not Gaussian but determined by voltage and temperature. In that case, we define a parameter z_4 by $\langle\cos(2(\phi+\delta\phi))\rangle = z_4\cos(2\phi)$. Here it is understood that $\langle\delta\phi\rangle = 0$ (so ϕ corresponds to the average phase), and we have $z_4 = z^4$ for the Gaussian case. In Eq. (17) the factor z^2 in front of $\cos(2\phi)$ changes to $(z_4 - z^2)/(z^2 - 1)$. For example, at $T = 0$ we have to average over a box distribution of width $\delta k = eV/(\hbar v_F)$, which yields $z = \sin(\delta k\delta x)/(\delta k\delta x)$and $z_4 = 2\sin(\delta k\delta x/2)/(\delta k\delta x)$ (compare the discussion in Section II). Hence the phase-averaged shot noise $\langle T_1T_2\rangle$ can still depend on the average phase ϕ even when the visibility is zero ($z = 0$, $z_4 \neq 0$ for $\delta k\delta x = (2n+1)\pi$), in marked contrast to dephasing or Gaussian phase fluctuations.

If we use the extra terminal φ to model inelastic relaxation[16] instead of pure dephasing, its distribution function f_φ is given by an equilibrium Fermi function of appropriate chemical potential, and the only condition is that the energy-integrated current must vanish at each instant of time (voltage probe). This implies that the chemical potential at this reservoir fluctuates. It turns out that in the inelastic case it does matter whether relaxation is ascribed fully to one arm or to both arms. Therefore, we set up a model with reservoirs L, R with associated amplitudes z_L, z_R. As the current only depends on $z_Lz_R \equiv z$, we write $z_L = z^\lambda$ and $z_R = z^{1-\lambda}$, where the parameter λ quantifies the asymmetry ($\lambda = 1, 0$ for relaxation in the left/right arm and $\lambda = 1/2$ for the symmetric case). In evaluating the shot noise at terminal 3 we have to take into account the current correlations between terminals 3, L and R, along the same lines as before. The expression in Eq. (16) is replaced by:

$$\langle T_1\rangle\langle T_2\rangle - 2R_AT_BR_B(1+(1-2T_A)z^2 - R_A(z^{2(1-\lambda)}+z^{2\lambda})), \quad (18)$$

for $R_A < T_A$ (otherwise interchange R_A, T_A). In the fully asymmetric case ($\lambda = 0, 1$), we recover the result (16) obtained for pure dephasing. However, in general the shot noise may be reduced: For example, at $\lambda = 1/2$ and

$T_A = 1/2$ Eq. (18) turns into $\langle T_1 \rangle \langle T_2 \rangle - R_B T_B (1-z)$, which can become zero even in the limit of full relaxation ($z=0$), at $T_B = 1/2$ (see Fig. 3).

For reference purposes, we also list the generalization of the pure dephasing result, Eq. (16), to the case of finite temperatures and energy-dependent transmission probabilities:

$$\frac{2\pi S_{33}}{e^2} = \int dE\, (\delta f \langle T_1 \rangle + f)(1 - (\delta f \langle T_1 \rangle + f)) \\ + f(1-f) - 2(1-z^2) R_A R_B T_A T_B \delta f^2 \quad (19)$$

Here $f = f_2 = f_3$ is a thermally smeared Fermi function, and $\delta f(E) = f(E - eV) - f(E)$ is the difference of distributions in reservoirs 1 and 2.

It should be emphasized that the phenomenological dephasing terminal approach cannot yield the dependence of dephasing strength on voltage and temperature, since the strength of dephasing, z, enters as an arbitrary parameter. It also does not account for the spectrum of environmental fluctuations, which is important to provide a smooth cross-over between dephasing and phase averaging if the ratio of the typical fluctuation frequencies to other characteristic frequencies (e.g. voltage and temperature) is varied (see Section VII).

In this section, we have calculated the effect of pure dephasing on shot noise in a mesoscopic Mach-Zehnder interferometer for electrons, using the scattering theory of shot noise and the dephasing terminal approach. The resulting shot noise expression is, in general, different from a simple phase average of the usual partition noise formula, and therefore may be employed to distinguish real dephasing from mere phase averaging. However, the result also differs from what one might obtain by merely inserting transmission probabilities where the effect of dephasing has already been taken into account. We have pointed out that dephasing and phase averaging become indistinguishable in the limit of zero visibility (within this model), but that a strong difference may be observed for other parameter values.

VI. POSSIBLE SHORTCOMING OF THE DEPHASING TERMINAL

In this section, we reexamine the difference between the shot noise results obtained from the simple classical model of Section IV and the dephasing terminal of the previous section. We will take the classical model as our starting point and investigate how the dephasing terminal approach would be implemented within the context of this model. As we will see, the extra suppression of shot noise in the dephasing terminal turns out to be artificial.

For simplicity, we restrict attention to the case of a symmetric first beam splitter, $T_A = 1/2$. We now focus on the second beam splitter B and ask for the shot

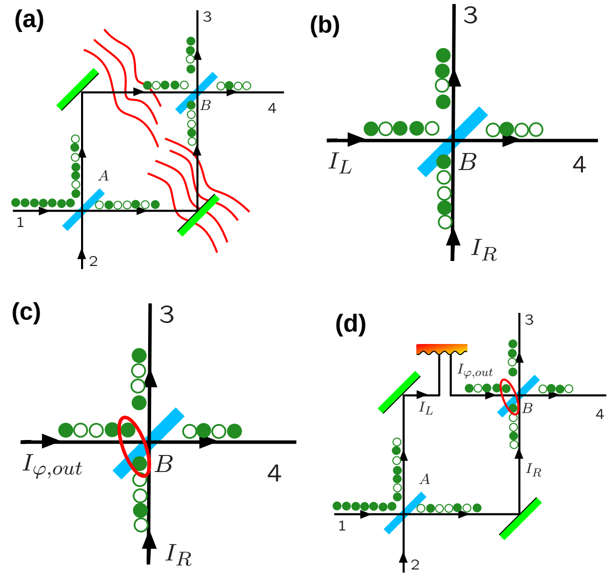

Figure 4: Interpretation of the dephasing terminal within the context of the simple classical model (a): The shot-noise reduction due to anticorrelations (b) and the Pauli principle (c) are kept at the same time in the dephasing terminal approach (d).

noise at its output port. The initial classical model (see Fig. 4a) leads to perfectly anticorrelated streams of electrons in the left and right arm, entering beam splitter B (see Fig. 4b). Thus we can obtain the correct result by treating the inputs as two incoherent, but completely anticorrelated sources. We have

$$(I_L,\, I_R) = (1,0) \,\text{or}\, (0,1), \quad (20)$$

each with probability 1/2 (in every "elementary timestep"). $\langle I_3 \rangle$ and $\langle \delta I_3^2 \rangle$ give the same result as before, Eq. (6).

We will now apply the dephasing terminal calculation to our simplified model, see Fig. 4d.

Following closely the procedure of the previous section, we will at first calculate the shot noise at the output port without taking into account the fluctuations of the distribution function in the dephasing terminal. This corresponds to the calculation of the "intrinsic" current fluctuations δI. In the previous section, this was done using the scattering theory of shot noise, and it will be performed on the basis of classical probability theory in this section. It means the two inputs to beamsplitter B are treated as uncorrelated sources of electrons (see Fig. 4c). At first sight we expect this to give different results than before, possibly with an increased shot noise at the output ports, as the shot-noise supression due to anticorrelations is lifted. According to this expectation, accounting for the anticorrelations (by way of the fluctuating distribution function) would then decrease the shot noise result and ultimately give the "correct" answer, ob-

tained in the previous paragraph. Nevertheless, this will turn out *not* to be the case, the shot noise *without* anticorrelations will turn out to be the same as before (and the subsequent introduction of anticorrelations will spoil the agreement with the correct result).

The complete model is now described by:

$$(I_L, I_R) = (1, 0) \text{ or } (0, 1), \tag{21}$$

each with probability 1/2 in every timestep, and, *independently*,

$$I_{\varphi,\text{out}} = 1 \text{ or } 0, \tag{22}$$

with probability 1/2, for the current entering the second beam splitter from the left arm (i.e. after the dephasing terminal).

The average current is 1/2, as before. However, we have to be careful when calculating the shot noise, as two electrons might impinge simultaneously onto B (ellipse in Fig. 4c), in which case a classical treatment would permit both to go into the same output port (with probability $T_B R_B$ in the present model), while in reality the Pauli principle prevents them from doing so. We find the following table of probabilities, each line occuring with probability 1/4:

$I_{\varphi,out}$	I_R	$P(I_3=0)$	$P(I_3=1)$	$P(I_3=2)$
0	0	1	0	0
1	0	T_B	R_B	0
0	1	R_B	T_B	0
1	1	0	1	0

From this, we obtain

$$\left\langle \delta I_3^2 \right\rangle = \frac{1}{4}\,, \tag{23}$$

which happens to be identical to the result calculated for anticorrelated inputs. If, however, we had neglected the Pauli principle, we would have obtained a larger shot noise,

$$\left\langle \delta I_3^2 \right\rangle = \frac{1}{4} + \frac{1}{2} R_B T_B \text{ (no Pauli principle).} \tag{24}$$

Therefore, the inclusion of the Pauli principle effects at the second beamsplitter has suppressed the shot noise by $R_B T_B/2$.

However, according to the logic of the dephasing terminal approach, we still have to ensure the total current into the dephasing terminal to vanish at each point in time. We will proceed as for the full dephasing terminal calculation of the preceding section, i.e. by postulating a fluctuating distribution function at the terminal which is chosen to compensate the fluctuations δI_φ that would be present otherwise. Although this will effectively (and correctly) re-introduce some anti-correlations between the two input currents to the beamsplitter B, the postulated relation between "intrinsic" current fluctuations δI_φ and corresponding distribution function fluctuations δf_φ (Eq. (9)) constitutes an ad hoc semiclassical *ansatz*. This is in contrast to the rest of the dephasing terminal approach, which just represents a valid model of a particular scattering geometry, designed to mimick some aspects of dephasing.

As a consequence, the full current fluctuations in the output port are changed (see Section V):

$$\Delta I_3 = \delta I_3 + R_B \delta I_\varphi \tag{25}$$

We will now calculate $\left\langle \Delta I_3^2 \right\rangle$ by taking the correlators $\langle \delta I_3 \delta I_\varphi \rangle$, $\left\langle \delta I_3^2 \right\rangle$ and $\left\langle \delta I_\varphi^2 \right\rangle$ from the underlying classical model, instead of the quantum mechanical scattering theory of shot noise. Since $\left\langle \delta I_3^2 \right\rangle = 1/4$ alone would give the correct result for the noise of the output current (see above), it is already clear at this point that any further contributions must lead to an artificial deviation from the correct value.

The total current into the dephasing terminal is

$$I_\varphi = I_L - I_{\varphi,\text{out}}, \tag{26}$$

which is forced to be zero at all times. Using this relation, as well as the probabilities prescribed above, we find

$$\left\langle \delta I_\varphi^2 \right\rangle = 1/2 \tag{27}$$

and

$$\langle \delta I_3 \delta I_\varphi \rangle = -1/4. \tag{28}$$

This finally gives:

$$\left\langle \Delta I_3^2 \right\rangle = \frac{1}{4} - \frac{1}{2} R_B T_B. \tag{29}$$

Therefore, the shot noise calculated with the help of the dephasing terminal ansatz is reduced (at $T_B \neq 0, 1$) as compared to what is found for the original model. The reason should have become transparent from our step-by-step derivation: The ansatz (25) serves to (correctly) take into account anticorrelations between the two inputs to beamsplitter B, but it does *not* throw out the Pauli principle effects that determine the shot noise result for two uncorrelated sources. In reality, only one effect or the other is present, while the dephasing terminal approach keeps both of them, thereby artificially reducing the shot noise. In the full calculation of Section V, the problem can be traced to the ansatz describing the fluctuations δf_φ of the distribution function as a fluctuating

c-number function of time. In that way, the dephasing terminal approach is no longer fully quantum-mechanical (in contrast to the calculation of the current itself, where δf_φ is not needed).

It is interesting to note that there is no problem if we assume the path-length difference between the two arms of the interferometer to be large ($eVv_F\delta x \gg 1$). We can incorporate this within the simple classical model by assuming there to be a time-lag between the anticorrelated input streams to the second beam splitter. Going through steps similar to those above, we find a shot-noise reduction even in the initial classical model, to a value given by Eq. (29), which is also the value found from the full dephasing terminal calculation for that limit. This fits perfectly with our reasoning from above: In this case, the anticorrelations and the effects due to the Pauli principle are indeed present at the same time.

In this section we have demonstrated that the ansatz used for shot noise calculations in presence of the dephasing terminal fails to give the correct result when applied to this simple model of incoherent transmission through a Mach-Zehnder interferometer. Moreover, the (artificially reduced) shot noise result is even identical to what is found in the more sophisticated calculation (Section V), where the dephasing terminal ansatz is the same but correlators are evaluated using the scattering theory of shot noise (instead of simple classical probabilities). Thus it is likely that this calculation is affected by the same problem, which artificially reduces shot noise (and makes it coincide with the phase-averaged shot-noise result).

Strictly speaking, we have not given a direct proof of the failure of the dephasing terminal ansatz for shot noise calculations, as we have been able to present a detailed analysis only by taking the heuristic classical model as our starting point. This has been necessary because we lack a simple quantum-mechanical version of the fully incoherent case, against which we should compare the results of the dephasing terminal. Although we do consider a microscopic model in the next section, the results obtained there cannot necessarily be compared with the dephasing terminal either, since the dephasing terminal represents a phenomenological model and it is unclear to which miscroscopic models it should correspond (if any). Nevertheless, the arguments of this section strongly suggest that the results of the dephasing terminal approach to shot noise should be treated with caution, at least for geometries similar to the two-way interferometer considered in this article.

VII. DEPHASING BY CLASSICAL NOISE

In this section, we will introduce a microscopic, fully quantum-mechanical model of dephasing, and derive the resulting current-noise. Its major advantages are that it displays the dependence of the results on the power spectrum of the environmental fluctuations (which cannot be done in any of the phenomenological models discussed above), that it may be related directly to microscopic fluctuations acting on the electrons, and that it properly treats all quantum-mechanical effects regarding the motion of electrons. A brief discussion of the model and some of the most important results has already been presented in Ref. 21.

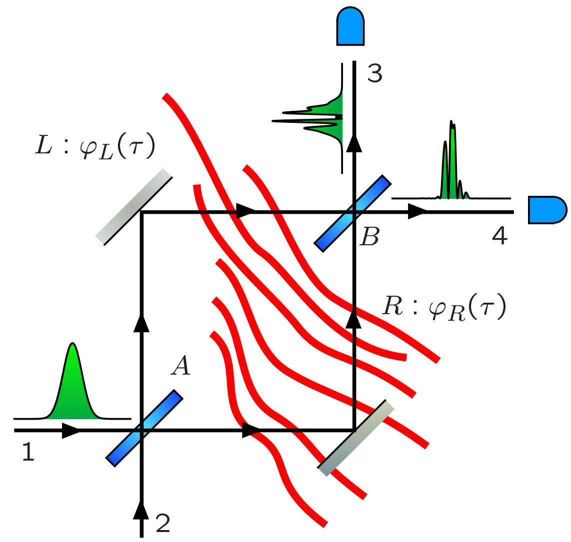

Figure 5: The Mach-Zehnder interferometer setup analyzed in the text. In the case shown here, the fluctuations of the environment are fast compared with the temporal extent of the wave packet (determined by temperature or voltage, see text). The probability density of the incoming wave packet and its two outgoing parts is shown.

There is one major simplifying feature of the model we are going to use: We will assume dephasing to be induced by the fluctuations of a *classical* potential $V(x,t)$, acting on the electrons as they traverse the interferometer (see Fig. 5). Classical noise may be used as an approximation to the effects of a truly quantum-mechanical environment (e.g. phonons, Nyquist noise due to other electrons in nearby gates). In fact, this approximation has been employed in the past, e.g. in the theory of dephasing in weak localization[26]. The idea is to use a classical fluctuating potential whose correlator is set equal to the symmetrized part of the quantum-mechanical correlation function. However, the zero-point fluctuations are (usually) to be omitted, since Golden-Rule type calculations suggest that their effect is canceled by the restriction of the scattering phase space via the Pauli principle. Nevertheless, this approximation of neglecting the environmental zero-point fluctuations can only be good as long as $eV \ll T$. Otherwise, the scattering phase space will be determined by the nonequilibrium Fermi functions in the arms of the interferometer, and thus this simple prescription must fail.

On the other hand, our model may also describe nonequilibrium (classical) microwave noise impinging onto the interferometer setup, or some thermal noise

source that behaves essentially classically (i.e. where $\omega < T$ for the relevant fluctuation frequencies). In this case, the treatment becomes exact and holds for all values of voltage and temperature.

As the noise is classical, we still have a (time-dependent) single-particle problem, i.e. we can solve for the motion of individual electrons. The Fermi function will enter only in the end, when expectation values (such as current correlators) are calculated. The Pauli principle does not enter the calculation (except for, possibly, the potential correlator, as explained above). In contrast, for the case of a fully quantum-mechanical environment, we would end up with a complicated many-body problem, since in any case the electrons would feel an effective interaction induced by the coupling to the bath - even if we were to neglect their intrinsic interaction in the beginning. This problem is deferred to a future analysis.

A. Electron field at output port

As the electrons travel along the interferometer arms, they will accumulate a random phase, due to the fluctuating potential. We neglect the additional effects of the potential, namely acceleration and decceleration, by assuming the electron's velocity to remain constant (linearized dispersion relation). This should be a good approximation for sufficiently large Fermi energy. The effects of a non-constant velocity have been analysed in more detail in Ref. 11, where Nyquist-noise induced dephasing of the current in a Mach-Zehnder setup has been studied using the WKB approximation (see also Ref. 14). There, the main contribution to the end-result for the dephasing rate did not depend on these extra effects. We will also assume backscattering to be absent (i.e. the electrons are traveling along chiral edge channels, or the potential is sufficiently smooth to prevent $2k_F$ momentum transfers). Finally, as we are taking a model of non-interacting electrons as our starting point, the electrons' spin does not play any important role (except for trivial factors), and we assume the electrons to be spin-polarized in the following.

The Heisenberg equation of motion for the electron field $\hat{\Psi}$ moving at constant velocity v_F, under the action of a fluctuating potential $V(x,t)$, reads:

$$i\partial_t \hat{\Psi}(x,t) = \left[\epsilon_F + v_F(-i\partial_x - k_F) + V(x,t)\right] \hat{\Psi}(x,t) . \tag{30}$$

Here x denotes the coordinate along the respective arm of the interferometer. By solving this equation of motion, and taking into account the action of the beamsplitters, we arrive at the following expression for the electron field at some point in the outgoing lead 3:

$$\hat{\Psi}(x,\tau) = \int \frac{dk}{\sqrt{2\pi}} e^{-i\epsilon_k \tau} \sum_{\alpha=1}^{3} t_\alpha(k,\tau) \hat{a}_\alpha(k) e^{s_\alpha i k_F x} \tag{31}$$

The field $\hat{\Psi}$ is a linear superposition of the electron fields $\hat{a}_\alpha$ emitted at the reservoirs $\alpha = 1,2,3$. Note that for the special case of chiral edge channels, we may choose to concentrate only on the outgoing current, such that $\alpha = 3$ would be absent from Eq. (31), and the corresponding trivial contributions to subsequent equations would drop out as well. We have $t_3 = 1$, $s_{1,2} = 1$, $s_3 = -1$, the reservoir operators obey $\left\langle \hat{a}_\alpha^\dagger(k) \hat{a}_\beta(k') \right\rangle = \delta_{\alpha\beta}\delta(k-k') f_\alpha(k)$ with f_α the distribution function in reservoir α, and the integration is over $k > 0$ only.

In contrast to the usual case, the transmission amplitudes t_α have become time-dependent. The amplitudes t_1, t_2 for an electron to go from terminal 1 or 2 to the output terminal 3 depend on fluctuating time-dependent phases $\varphi_{L,R}$:

$$\begin{aligned} t_1(k,\tau) &= t_A t_B e^{i\varphi_R(\tau)} + r_A r_B e^{i\varphi_L(\tau)} e^{i(\phi + k\delta x)} && (32) \\ t_2(k,\tau) &= t_A r_B e^{i\varphi_L(\tau)} e^{i(\phi + k\delta x)} + r_A t_B e^{i\varphi_R(\tau)} && (33) \end{aligned}$$

Here $t_{A/B}$ and $r_{A/B}$ are energy-independent transmission and reflection amplitudes at the two beamsplitters (with $t_j^* r_j = -t_j r_j^*$), δx accounts for a possible path-length difference between the interferometer arms, and ϕ denotes the Aharonov-Bohm phase due to the flux through the interferometer. The electron accumulates fluctuating phases while moving along the left or right arm:

$$\varphi_{L,R}(\tau) = -\int_{-\tau_{L,R}}^{0} dt'\, V(x_{L,R}(t'), \tau + t') \,, \tag{34}$$

where τ is the time when the electron leaves the second beamsplitter after traveling for a time $\tau_{L,R}$ along the interferometer arms, the trajectories being described by $x_{L,R}(t)$.

In our model, the total traversal times $\tau_{L,R}$ enter only at this point, determining the relation between the phase correlator and the potential correlator. Note that we have assumed the interaction to be confined to the interferometer region. This assumption is natural if the fluctuations are due to gates or other localized disturbances. It is also sufficient for short-wavelength fluctuations. However, in the case of long-wavelength fluctuations, it means that the effect of these fluctuations on the phase difference $\varphi_L - \varphi_R$ will cancel out only in the case of vanishing path-length-difference. Otherwise, cutting off the potential V at the entry and exit beamsplitter automatically introduces some remaining fluctuations in $\varphi_L - \varphi_R$.

In general, the form of the phase correlator can be related to the potential correlator $\langle VV \rangle$, using Eq. (34). For abbreviation, we set $V_L(t_1,\tau) \equiv V(x_L(t_1), \tau + t_1)\theta(-t_1)\theta(t_1 + \tau_L)$ and likewise for V_R. Then we have $\varphi_L(\tau) = -\int dt' V_L(t', \tau + t')$ and thus:

$$\begin{aligned} \langle \delta\varphi(\tau)\delta\varphi(0) \rangle = \int dt_1 dt_2 \, \langle (V_L(t_1,\tau) - V_R(t_1,\tau)) \, \times \\ (V_L(t_2,0) - V_R(t_2,0)) \rangle \, . \end{aligned} \tag{35}$$

The terms of the type $\langle V_L V_L \rangle$ and $\langle V_R V_R \rangle$ describe phase fluctuations within the two arms separately, while the cross-terms $\langle V_L V_R \rangle$ will serve to suppress dephasing in the case of long-wavelength fluctuations. In a diagrammatic treatment of dephasing (e.g. in weak localization), the cross-terms would correspond to "vertex contributions", whereas the former relate to "self-energy terms".

In general, the potential correlator $\langle VV \rangle_{q\omega}$ and the corresponding phase correlator (Eq. 35) depend on the microscopic environment under consideration (cf. Ref. 11 for a calculation of spatially homogeneous potential fluctuations in the interferometer arms, due to Nyquist noise), as well as the geometry. A discussion of the potential and phase fluctuations for realistic microscopic dephasing mechanisms will be provided in a future work. Here we take the position that the phase correlator is given, and we want to obtain the consequences for the current noise.

B. Current

Given the expression for the electron field, it is now in principle straightforward to calculate the current and its correlators. In calculating these quantities, we have to take both a quantum-mechanical expectation value, as well as an average over the random process $V(x,t)$, or rather $\varphi_{L,R}$. This average will be denoted by $\langle \cdot \rangle_\varphi$ in the following. The output current,

$$\hat{I}(\tau) = e\hat{\Psi}^\dagger(x,\tau)\frac{-i\partial_x}{2m}\hat{\Psi}(x,\tau) + h.c., \tag{36}$$

follows from (31). We will set $x=0$, as well as $\epsilon_{k'} - \epsilon_k = v_F(k'-k)$:

$$\begin{aligned}\hat{I}(\tau) &= \frac{ev_F}{4\pi}\int dk dk'\, e^{i(\epsilon_{k'}-\epsilon_k)\tau}\left[\sum_\alpha t_\alpha(k,\tau)\hat{a}_\alpha(k)\right]^\dagger \\ &\quad \times \sum_\beta s_\beta t_\beta(k',\tau)\hat{a}_\beta(k') + h.c. \end{aligned}\tag{37}$$

Therefore, we have:

$$\begin{aligned} I &= \left\langle \left\langle \hat{I} \right\rangle \right\rangle_\varphi = \\ &\frac{ev_F}{2\pi}\int dk \left\{ -f_3(k) + \sum_{\alpha=1,2} f_\alpha(k)\left\langle T_\alpha(k)\right\rangle_\varphi \right\}. \end{aligned}\tag{38}$$

The current depends on the phase-averages of transmission probabilities $T_1 = |t_1|^2$ and $T_2 = 1 - T_1$:

$$\langle T_1 \rangle_\varphi = T_A T_B + R_A R_B + 2z\,(r_A r_B)^* t_A t_B \cos(\phi + k\delta x), \tag{39}$$

The interference term is suppressed by the factor $z \equiv \left\langle e^{i\delta\varphi}\right\rangle_\varphi$, where $\delta\varphi = \varphi_L - \varphi_R$. In writing down this expression, we have assumed $V(x,t)$ and thus $\delta\varphi$ to be distributed symmetrically around 0, such that $\langle \sin(\delta\varphi)\rangle_\varphi = 0$. For the special case of Gaussian statistics (which we will assume below), we have $z = \exp(-\left\langle \delta\varphi^2 \right\rangle/2)$. The factor z decreases the visibility of the interference pattern observed in $I(\phi)$, and it has been defined to correspond precisely to the phenomenological z introduced for the dephasing terminal model (see Eq. (7)). An additional suppression of the interference term may be brought about by the k-integration in Eq. (38), if $T\delta x/v_F > 1$ or $eV\delta x/v_F > 1$. With respect to the current, it is indistinguishable from dephasing, which provides the motivation of looking at shot noise in this context (see Section III).

We note that the current itself is independent of the spectrum of environmental fluctuations, as it only depends on the probability distribution of $\delta\varphi$ at any given moment (and not its time-dependent correlator). This will change when we look at shot noise. It would also be different for the case of a quantum-mechanical environment, where the "effective" spread of $\delta\varphi$ would depend on the part of the bath spectrum that is still active in dephasing, despite Pauli blocking.

C. Noise power: General formula

Before we turn to the calculation of the (zero-frequency) current noise power S, we briefly list the main ingredients that we will find below:

- A "classical current noise" $S_{\rm cl}$, which is due to the time-dependent fluctuations of the interferometer's conductance. The resulting current fluctuations are linear in the applied voltage, such that the corresponding noise power is quadratic in V.

- For any fixed external noise power, there is a finite current noise contribution $S_{\rm V=0}$ even at $V = 0$ and $T = 0$, due to the nonequilibrium radiation impinging on the system.

- The remainder of the full current noise contains the usual quantum-mechanical partition noise $\mathcal{T}(1-\mathcal{T})$, which will be modified due to the presence of the dephasing potential. The form of this modification depends on whether the fluctuations of the environment are "fast" or "slow" as compared to the time-scales set by voltage and temperature.

The full current noise power S can be split into two parts, by rewriting the irreducible current correlator:

$$\begin{aligned} S &= \int d\tau \left\langle \left\langle \hat{I}(\tau)\hat{I}(0)\right\rangle \right\rangle_\varphi - \left\langle \left\langle \hat{I}(0)\right\rangle \right\rangle_\varphi^2 = \\ &\int d\tau \left\langle \left\langle \hat{I}(\tau)\right\rangle \left\langle \hat{I}(0)\right\rangle \right\rangle_\varphi - \left\langle \left\langle \hat{I}(0)\right\rangle \right\rangle_\varphi^2 + \\ &\int d\tau \left\langle \left\langle \hat{I}(\tau)\hat{I}(0)\right\rangle - \left\langle \hat{I}(\tau)\right\rangle \left\langle \hat{I}(0)\right\rangle \right\rangle_\varphi \end{aligned}\tag{40}$$

The first integral on the r.h.s. describes shot noise due to the temporal fluctuations of the conductance, i.e. fluctuations of a classical current $I(\tau) = \left\langle \hat{I}(\tau) \right\rangle$ depending on time-dependent transmission probabilities. We denote its noise power as $S_{\rm cl}$. It rises quadratically with the total current, as is known from $1/f$-noise in mesoscopic conductors[27].

We now focus on the second integral, which will contain the modified partition noise (among other contributions, such as a finite "Nyquist noise" $S_{V=0}$). It is evaluated by inserting (31) and applying Wick's theorem (similar formulas appear in Ref. 28):

$$\left\langle \left\langle \hat{I}(\tau)\hat{I}(0) \right\rangle - \left\langle \hat{I}(\tau) \right\rangle \left\langle \hat{I}(0) \right\rangle \right\rangle_{\varphi} = \left(\frac{ev_F}{2\pi} \right)^2 \int dk dk' \sum_{\alpha,\beta=1,2,3} f_\alpha(k)(1-f_\beta(k'))\, K_{\alpha\beta}(\tau) e^{i(\epsilon_{k'}-\epsilon_k)\tau}. \qquad (41)$$

Here $K_{\alpha\beta}$ is a correlator of four transmission amplitudes. We have $K_{33} = 1$, $K_{3\alpha} = K_{\alpha 3} = 0$, and

$$K_{\alpha\beta}(\tau) \equiv \left\langle t^*_\alpha(k,\tau) t_\beta(k',\tau) t_\alpha(k,0) t^*_\beta(k',0) \right\rangle_\varphi , \qquad (42)$$

for $\alpha, \beta = 1, 2$.

D. Limiting cases

In order to understand the resulting expressions, we will now derive two limiting forms, for a "fast" and a "slow" environment. We will assume that the phase correlator $\langle \delta\varphi(\tau)\delta\varphi(0) \rangle$ decays on some time-scale τ_c, the correlation time of the environment. Note that even for a non-exponential decay we can still define a typical scale τ_c, e.g. by demanding $\langle \delta\varphi(\tau_c)\delta\varphi(0) \rangle = \left\langle \delta\varphi^2 \right\rangle /2$. Now this time has to be compared against the other timescales, $(eV)^{-1}$ and T^{-1}. These scales enter the current noise formula (41) in the form of the Fermi functions, and they determine the τ-range of the oscillating exponential factor, after integration over k and k'. We will assume for the moment that the k-dependence of $K_{\alpha\beta}$ itself is unimportant (i.e. δx is sufficiently small), see below for a discussion of other cases.

For $eV\tau_c \ll 1$ and $T\tau_c \ll 1$ ("fast environment"), the major contribution of the integration comes from $|\tau| \gg \tau_c$, where $K_{\alpha\beta}$ factorizes into

$$K_{\alpha\beta}(\tau) \approx K_{\alpha\beta}(\infty) \equiv \left| \langle t^*_\alpha(k,0) t_\beta(k',0) \rangle_\varphi \right|^2 . \qquad (43)$$

Adopting this limiting value for $K_{\alpha\beta}$ at all times τ yields the noise power

$$\frac{S_{\rm fast}}{e^2 v_F/2\pi} = \int dk \sum_{\alpha,\beta=1,2} f_\alpha(1-f_\beta) \left| \langle t^*_\alpha t_\beta \rangle_\varphi \right|^2 + f_3(1-f_3), \qquad (44)$$

where we have set $f_{\alpha,\beta} = f_{\alpha,\beta}(k)$ and $t_{\alpha,\beta} = t_{\alpha,\beta}(k,0)$. Note that this form of the shot noise for a "fast" environment is not equivalent to an expression of the kind $\langle \mathcal{T} \rangle_\varphi (1 - \langle \mathcal{T} \rangle_\varphi)$, which we have obtained from a simple classical model (see the discussion in Section IV). The difference between those two formulas can be evaluated in general, and we find:

$$\left| \langle t^*_1 t_2 \rangle_\varphi \right|^2 - \langle T_1 \rangle_\varphi (1 - \langle T_1 \rangle_\varphi) = (z^2-1) R_B T_B . \qquad (45)$$

This means the partition noise for the "fast" case is usually reduced below the value found from the simple expression. Nevertheless, we will discuss a certain special case where the simple formula is indeed recovered, see below.

We can always write the full noise power as

$$S = S_{\rm fast} + S_{\rm fluct} + S_{\rm cl} , \qquad (46)$$

where $S_{\rm fluct}$ denotes the remainder besides $S_{\rm fast}$ and $S_{\rm cl}$, i.e. $S_{\rm fluct}$ is given by Eq. (41)) with $K_{\alpha\beta}(\tau) - K_{\alpha\beta}(\infty)$ inserted in place of $K_{\alpha\beta}(\tau)$. It yields a contribution to the Nyquist noise $S_{V=0}$ (see below), but apart from that it becomes important only at larger V, T, where it will serve to produce the crossover to the case of the "slow" environment, which we discuss now.

In the other limiting case the τ-integration is dominated by $|\tau| \ll \tau_c$ ("slow environment"), and we can use $K_{\alpha\beta}(\tau) \approx K_{\alpha\beta}(0)$, which yields

$$\frac{S_{\rm slow}}{e^2 v_F/2\pi} = \int dk \, \langle (f_1 T_1 + f_2 T_2)(1 - (f_1 T_1 + f_2 T_2)) \rangle_\varphi + f_3(1-f_3), \qquad (47)$$

i.e. the phase-average of the usual shot noise expression (at $T = 0$ the expression in brackets reduces to $\langle T_1(1-T_1) \rangle_\varphi$).

E. Evaluation of shot noise contributions in general

For a phase difference $\delta\varphi$ described by a Gaussian random process of zero mean and prescribed correlator

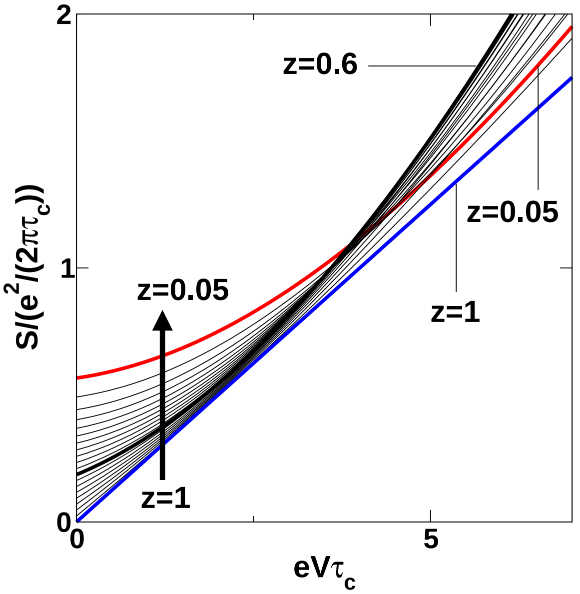

Figure 6: Full current noise S as a function of $eV\tau_c$, for increasing strength of dephasing ($z = 1 \dots 0.05$), according to Eq. (55). Dephasing always increases the current noise beyond the value obtained for the ideal case $z = 1$. The offset $S_{V=0}$ is given in Eq. (60), the slope near $V = 0$ is described by S_{fast} , Eq. (44), and at higher voltages the dependence on V is quadratic, due to S_{cl}. When S_{cl} is subtracted, the slope at large $eV\tau_c$ is determined by S_{slow} (i.e. $\langle T_1(1-T_1)\rangle_\varphi$). Parameters: $T = 0$, $\delta x = 0$, $\phi = \pi/2$, $T_A = 1/2$, $T_B = 0.3$.

$\langle \delta\varphi(\tau)\delta\varphi(0)\rangle$, the correlators $K_{\alpha\beta}$ of transmission amplitudes (Eq. (42)) can be evaluated in general. This is done by inserting the transmission amplitudes given above, and evaluating the average of the exponential phase factors. We thus obtain an exact expression which contains arbitrary orders of interaction with the field (i.e. arbitrary powers of the phase correlator).

The following expressions describe the time-dependent deviation of the transmission amplitude correlators $K_{\alpha\beta}(\tau)$ from their large-time limiting values $K^\infty_{\alpha\beta}$ (entering S_{fast}, Eq. (44)) . They follow directly from the definition of $K_{\alpha\beta}(\tau)$, Eq. (42), using the transmission amplitudes in Eqs. (32) and (33), as well as the abbreviations $g(\tau) = \exp\langle \delta\varphi(\tau)\delta\varphi(0)\rangle$ and $z = \exp(-\langle \delta\varphi^2\rangle/2)$:

$$\begin{aligned} K_{12}(\tau) - K^\infty_{12} = K_{21}(\tau) - K^\infty_{21} = \\ -2R_A T_A R_B T_B \cos(2\phi + \delta x(k+k'))z^2\left[g^{-1}(\tau) - 1\right] \\ +R_B T_B (R_A^2 + T_A^2) z^2 \left[g(\tau) - 1\right] \end{aligned} \quad (48)$$

$$\begin{aligned} K_{11}(\tau) - K^\infty_{11} = K_{22}(\tau) - K^\infty_{22} = \\ 2R_A T_A R_B T_B z^2 \\ \times \left\{\cos(2\phi + \delta x(k+k'))\left[g^{-1}(\tau) - 1\right] + \left[g(\tau) - 1\right]\right\} \end{aligned} \quad (49)$$

Here $R_A = 1 - T_A$, and we have repeatedly used the fact that there is a phase shift of $\pm\pi/2$ between transmission and reflection amplitudes at each beamsplitter ($r_A t^*_A = -r^*_A t_A$).

Both S_{fluct} and S_{cl} depend on the frequency spectrum of the environment via the exponential $g(\tau)$ of the phase correlator appearing in $K_{\alpha\beta}$ (in contrast, S_{fast} and S_{slow} are expressed in terms of $z = \exp(-\langle \delta\varphi^2\rangle/2)$ only). The resulting noise power can be written in terms of the following Fourier transforms (with $n = \pm 1$):

$$\hat{g}_n(\omega) \equiv \int d\tau\, e^{i\omega\tau}[e^{n\langle \delta\varphi(\tau)\delta\varphi(0)\rangle} - 1]. \quad (50)$$

Note that the first term in brackets approaches 1 for $|\tau| \to \infty$, as the phase correlations decay. These functions are similar to those appearing in the so-called "P(E)-theory" of tunneling in a dissipative environment[29,30] as well as in the "independent boson model".

Using the explicit forms of the correlators $K_{\alpha\beta}$, we find S_{fluct} to be equal to:

$$\begin{aligned} S_{\text{fluct}} = \left(\frac{ev_F}{2\pi}\right)^2 \int dkdk'\, [f_1(1-f'_2) + f_2(1-f'_1)] \\ \times R_B T_B \left\{(R_A^2 + T_A^2) z^2 \hat{g}_+(v_F(k'-k)) - \right. \\ \left. 2\cos(2\phi + \delta x(k+k')) R_A T_A z^2 \hat{g}_-(v_F(k'-k))\right\} + \\ [f_1(1-f'_1) + f_2(1-f'_2)] \\ \times 2z^2 R_A T_A R_B T_B \left\{\hat{g}_+(v_F(k'-k)) + \right. \\ \left. \cos(2\phi + \delta x(k+k'))\hat{g}_-(v_F(k'-k))\right\} \end{aligned} \quad (51)$$

In a similar fashion, we can evaluate S_{cl}. This term does not display two different limiting regimes. The reason is that it involves only correlators of time-dependent transmission probabilities, but no oscillating factor depending on the energy difference. Therefore, the result does not depend on the relation between τ_c and eV, T. In general, this term is determined by the zero-frequency correlators of the exponential phase factors contained in the transmission probabilities. We find (with $\delta f \equiv f_1 - f_2$):

$$\begin{aligned} S_{\text{cl}} = 2z^2 R_A R_B T_A T_B \left(\frac{e}{2\pi}\right)^2 v_F^2 \int dkdk'\, \delta f \delta f' \\ \times [\hat{g}_-(0)\cos(2\phi + \delta x(k+k')) + \hat{g}_+(0)\cos(\delta x(k-k'))] \end{aligned} \quad (52)$$

F. Current noise at $T = 0$

It still remains to evaluate the k-integrals contained in the expressions (51),(52) for S_{fluct} and S_{cl}. In this section, we will present and discuss explicit expressions for the case $T = 0$, $\delta x eV/v_F \ll 1$, i.e. the case of pure dephasing without any thermal smearing. According to the discussion at the beginning of the present section, analyzing the zero-temperature limit invariably means we adopt the picture of real classical noise impinging onto

the system (as opposed to classical noise being an approximation for a quantum bath, which would require $eV \ll T$ for selfconsistency).

We assume the electrons to be injected from reservoir 1, i.e. $f_2(k) = f_3(k) = f(k) = \theta(k_F - k)$ and $f_1(k) = \theta(k_F + \Delta k - k) \equiv f(k) + \delta f(k)$, with $\Delta k = eV/v_F$.

As we are interested in the *shot noise*, we subtract the equilibrium part $S_{\text{fluct}}(V=0) \equiv S_{V=0}$ from S_{fluct} (Eq. (51)). In the remainder, the term stemming from $f_1(1-f_1')$ is seen not to contribute (employing symmetry in k and k'), and the terms from $f_1(1-f_2')$ and $f_2(1-f_1')$ lead to the integral

$$\int dkdk' \left(\delta f(1-f') - f\delta f'\right) \hat{g}_n(v_F(k'-k)) = \frac{2eV}{v_F^2} I_n(V), \qquad (53)$$

where $I_n(V)$ also depends on temperature T. In particular, at $T=0$, we find I_n to be:

$$I_n(V) \equiv \int_0^{eV} d\omega\, (1 - \frac{\omega}{eV})\hat{g}_\lambda(\omega) \qquad (54)$$

Collecting the contributions from $S = S_{\text{cl}} + S_{\text{fast}} + S_{\text{fluct}}$, the shot noise is then given by:

$$\begin{aligned}\frac{S - S_{V=0}}{e^3V/2\pi} &= \frac{eV}{\pi} z^2 R_A R_B T_A T_B (\cos(2\tilde{\phi})\hat{g}_-(0) + \hat{g}_+(0)) \\ &+ \left|\langle t_1^* t_2\rangle_\varphi\right|^2 \\ &+ \frac{1}{\pi} z^2 R_B T_B \left\{-2\cos(2\tilde{\phi}) R_A T_A\, I_-(V)\right. \\ &\left. + (R_A^2 + T_A^2)\, I_+(V)\right\} \end{aligned} \qquad (55)$$

Here we have defined the average phase as $\tilde{\phi} = \phi + k_F \delta x$. The first line of Eq. (55) corresponds to S_{cl}, the second to S_{fast}, and the rest to $S_{\text{fluct}} - S_{V=0}$. The current noise displayed in Eq. (55) is a function of $eV\tau_c$, z, T_A, T_B, ϕ, and of the detailed shape of the environment correlator contained in $I_n(V)$ and $\hat{g}_n(0)$. The dependence of $S - S_{V=0}$ on voltage is explicit in the first two lines, stemming from S_{cl} and S_{fast} (quadratic and linear, respectively). Only the contribution from S_{fluct} (last two lines) depends on voltage in a more complicated way, via the environment spectrum.

We can introduce the dependence on the environment correlation time τ_c by assuming the phase-correlator to be given as $\langle \delta\varphi(\tau)\delta\varphi(0)\rangle = C(\tau/\tau_c)$. Then $I_n(V)$ is a function of $eV\tau_c$ only.

We may confirm directly that S_{fast} dominates at low voltages, since S_{cl} is quadratic in voltage and the integrals $I_\pm(V)$ in S_{fluct} vanish. At large $eV\tau_c \gg 1$ we can use the sum-rule

$$I_n(V) \to \pi\left[z^{-2n} - 1\right] \qquad (56)$$

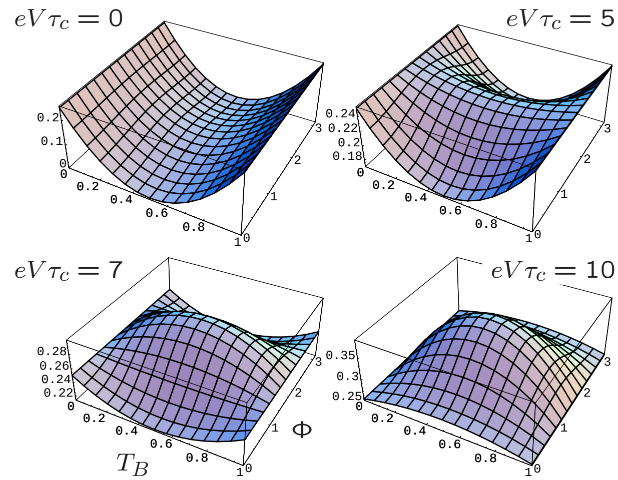

Figure 7: Normalized shot noise $(S - S_{V=0})/(e^3V/2\pi)$ of the Mach-Zehnder interferometer, as a function of the transmission of the second beamsplitter, T_B (horizontal axis), and the phase difference ϕ (into the plane), for the case of small but finite visibility, $z = 1/e$, at $T = \delta x = 0$ and $T_A = 1/2$. The different plots show the succession from a "fast" environment to a "slow" one, by increasing the voltage or the correlation time (top left to bottom right: $eV\tau_c = 0, 5, 7, 10$). Note the change of plot range on the vertical axis. At $T_B = 0, 1$, the normalized shot noise remains fixed at 1/4.

to combine the shot noise contributions in the last three lines of Eq. (55), i.e. $S_{\text{fast}} + S_{\text{fluct}} - S_{\text{V=0}}$, yielding:

$$\begin{aligned}&\left|\langle t_1^* t_2\rangle_\varphi\right|^2 - 2\cos(2\tilde{\phi}) R_A T_A R_B T_B\, z^2(z^2-1) \\ &+ R_B T_B (R_A^2 + T_A^2)\,(1 - z^2) = \langle T_1(1-T_1)\rangle_\varphi \end{aligned} \qquad (57)$$

This is precisely the result expected from the limit of a "slow" bath, i.e. from S_{slow}, compare Eq. (47). At intermediate voltages, the shot noise interpolates smoothly between the extremes described by S_{fast} and S_{slow}.

To produce the plots discussed in the following, we have assumed 50% transparency of the first beamsplitter ($T_A = 1/2$) and a simple Gaussian form for the phase correlator,

$$\langle \delta\varphi(\tau)\delta\varphi(0)\rangle = \left\langle \delta\varphi^2\right\rangle e^{-(\tau/\tau_c)^2}. \qquad (58)$$

In the case of $T_A = 1/2$, the normalized shot noise is given explicitly as a function of the parameters $z, eV\tau_c, T_B, \phi$ by the following formula:

$$\begin{aligned}\frac{S - S_{V=0}}{e^3V/2\pi} &= \frac{z^2}{4\pi}(eV\tau_c) R_B T_B (\cos(2\tilde{\phi})\,\hat{\tilde{g}}_-(0) + \hat{\tilde{g}}_+(0)) + \\ &+ \frac{1}{4}\left[(T_B - R_B)^2 + 4z^2 R_B T_B \sin^2\tilde{\phi}\right] \\ &+ \frac{z^2}{2\pi} R_B T_B [\tilde{I}_+(V\tau_c) - \cos(2\tilde{\phi})\tilde{I}_-(V\tau_c)] \end{aligned} \qquad (59)$$

Here the functions $\tilde{\hat{g}}_n$ and $\tilde{I}_n$ are evaluated by setting $\tau_c = 1$ in the phase correlator $C(\tau/\tau_c)$. They have to be evaluated by numerical integration (for a given shape $C(\tau/\tau_c)$ of the phase correlator). This equation has been derived for the case $eV > 0$, but it may be verified that S is symmetric in V.

The full current noise S also contains the Nyquist noise $S_{V=0}$, which is independent of $\tilde{\phi}$ and T_A:

$$S_{V=0} = \frac{e^2}{2\pi^2} z^2 R_B T_B \int_0^\infty d\omega\, \omega \hat{g}_+(\omega)\,. \tag{60}$$

The Nyquist noise scales like $1/\tau_c$. The dependence on z is not explicit, as the integral depends on z itself (scaling like $1/z^2$ for small but not ultrasmall z). In deriving the Nyquist noise from S_{fluct}, we have only kept the contribution from states near the Fermi edge, assuming all states for $k \in (-\infty, k_F)$ to be filled.

Fig. 6 shows the evolution of $S(V)$ with increasing dephasing strength (i.e. increasing $\langle \delta\varphi^2 \rangle$, decreasing z). Note that the shot noise itself (i.e. the deviation from $V = 0$) may even vanish due to the presence of the fluctuating potential, in the limit of a "fast" environment, $V\tau_c \to 0$: According to Eq. (44), S_{fast} is determined by $\left|\langle t_1^* t_2 \rangle_\varphi\right|^2$ at $T = 0$. In the limit of vanishing visibility, $z \to 0$, this expression is zero for $T_B = 1/2$, independent of the value of T_A. That may be verified explicitly, but it can also be deduced from Eq. (45), by noting that $\langle T_1 \rangle_\varphi (1 - \langle T_1 \rangle_\varphi) = 1/4$ for $z = 0$, $T_B = 1/2$. However, although S_{fast} can become zero, the total current noise S does not vanish, due to the Nyquist contribution (and the classical term S_{cl} at higher voltages). Indeed the figure illustrates that the fluctuating potential $V(x,t)$ always leads to an increase in current noise (as expected). Nevertheless, the dependence on dephasing strength may be non-monotonic, as seen in Fig. 6, at large voltages V.

The dependence on $eV\tau_c$ is also illustrated in Fig. 7, where the dependence of the shot noise on the parameters T_B and ϕ is displayed for different values of $eV\tau_c$ (see also the figures in Ref. 21 showing the crossover between S_{fast} and S_{slow}).

Note that the behaviour of S_{fast}, given by $\left|\langle t_1^* t_2 \rangle_\varphi\right|^2$, is quite different from that of $\langle T_1 \rangle_\varphi (1 - \langle T_1 \rangle_\varphi)$, which is the form derived from the simple classical model of Section IV. Indeed, the latter expression does not vanish at intermediate values of T_A, T_B ($\neq 0, 1$), and for $z = 0$ it becomes independent of T_B if $T_A = 1/2$ (while the first expression becomes independent of T_A if $T_B = 1/2$).

G. Other cases: Finite temperatures and finite path-length difference

The results of the previous section have been derived for the case $T = 0$, $\delta x = 0$. We will now discuss the changes introduced by relaxing these assumptions.

Finite temperatures: If we calculate the current noise for a finite temperature T, but still at $\delta x = 0$, the different components of $S = S_{\text{cl}} + S_{\text{fast}} + S_{\text{fluct}}$ show the following behaviour: The contrast of the current $I(\phi)$ is unaffected by the thermal smearing of the Fermi surfaces (since $\delta x = 0$), and for the same reason the "classical" part S_{cl} remains the same (apart from possible changes related to a temperature-dependence of the environmental power spectrum). In S_{fast} from Eq. (44), the finite-temperature Fermi functions lead to Nyquist noise contributions (which have been absent in S_{fast} for $T = 0$):

$$\begin{aligned} \frac{S_{\text{fast}}}{e^2/2\pi} \;=\; & T\left\{\left|\langle T_1\rangle_\varphi\right|^2 + \left|\langle T_2\rangle_\varphi\right|^2 + 1\right\} + \\ & + eV\left|\langle t_1^* t_2\rangle_\varphi\right|^2 \coth\left(\frac{\beta eV}{2}\right) \end{aligned} \tag{61}$$

In S_{fluct} of Eq. (51) the k, k'-integral over products of Fermi functions and environment power spectra $\hat{g}_\lambda$ are altered as well. In the particular limit of $T\tau_c \to \infty$ (regardless of V), the Fermi functions can be approximated by a constant on the scale over which the power spectrum $\hat{g}_\lambda$ changes. Then the integrals over $k' - k$ can be carried out easily, leading to sum rules. Combining the terms from S_{fast} and S_{fluct} in this limit leads to the expression S_{slow} (Eq. (47)). We conclude that S_{slow} is indeed the appropriate expression for $1/\tau_c \ll \max(T, eV)$.

Finite path-length difference: If a finite path-length difference δx is introduced, we have to consider four time-scales altogether: τ_c, $(eV)^{-1}$, T^{-1} and the new time-scale $\delta x/v_F$. We will not give an exhaustive discussion of all possible cases for the order of these times. In the limiting case of very small δx, i.e. $\delta x/v_F \ll \tau_c, (eV)^{-1}, T^{-1}$, the previous expressions remain unchanged. Even if $\delta x/v_F$ becomes larger than τ_c (but remains much smaller than $(eV)^{-1}, T^{-1}$), it may still be shown that this does not affect the results for the current noise.

We now consider the more interesting opposite limit, where the averaging over wave-number k is so important that it destroys completely the interference pattern, i.e. $\delta x/v_F \gg (eV)^{-1}$ or $\delta x/v_F \gg T^{-1}$. In that case, the interference term in the average current is completely suppressed, such that the additional dephasing effect of the environment is unimportant for the current. In addition, the "classical" current noise part S_{cl} now vanishes, since it depends on the temporal fluctuation of the interference term in the average current $\left\langle \hat{I}(\tau) \right\rangle$, which is already absent due to thermal averaging. The other two parts S_{fast} and S_{fluct} of the current noise S are changed as well, but they do not become equal to the results obtained without dephasing.

We illustrate those changes in the zero-temperature case analyzed in section VII F. The shot noise in Eq. (59) is changed in the following ways: The first line (due to S_{cl}) is absent, and the second and third lines (due to S_{fast} and S_{fluct}) are averaged over the phase ϕ, such

that the average of $\cos(2\phi)$ vanishes and that of $\sin^2\phi$ is equal to $1/2$. However, the shot noise still depends on z and on the bath spectrum (via I_+):

$$\frac{S - S_{V=0}}{e^3V/2\pi} = T_A R_A (T_B - R_B)^2 + z^2 T_B R_B (T_A^2 + R_A^2)[1 + \frac{I_+(V)}{\pi}] \,. \quad (62)$$

For a "fast" environment, we have $I_+(V) \to 0$, such that Eq. (62) becomes $T_A R_A (T_B - R_B)^2$ in the fully incoherent case, $z \to 0$. In the opposite limit of large voltages ("slow environment", $eV\tau_c \gg 1$), we have $I_+(V) \to \pi\left[z^{-2} - 1\right]$, which makes Eq. (62) independent of z. The resulting expression is then equivalent to the one obtained by pure k-averaging, in the absence of dephasing. In conclusion, shot noise may indeed help to reveal the presence or absence of dephasing even when thermal averaging is so strong that interference is already completely suppressed, but not in the limit of a "slow" bath. The Nyquist noise is not affected by δx, since it results from setting $f_1 = f_2$ in Eq. (51), whence the cos-terms depending on δx combine to zero.

Beam of electrons: It is instructive to notice that even the dephasing model considered here can lead to the simple form $\langle T_1\rangle_\varphi (1 - \langle T_1\rangle_\varphi)$ of the shot-noise (which holds for a classical model in the fully incoherent limit, see Section IV). This is true provided the transport situation is different from the usual one treated above. Instead of having all the reservoirs filled up to some Fermi level and then applying a voltage between them, we consider a situation where a (nearly mono-energetic) beam of electrons is injected from reservoir 1, with wavenumbers in an interval Δk, and all the other reservoirs are *empty*: $f_1(k) = \theta(k \in [k_F, k_F + \Delta k]$ and $f_2 = f_3 = 0$. In this situation, there is no "Nyquist noise" (S vanishes for $\Delta k = 0$, when there are no electrons at all). In the limit of small Δk ("fast environment", with $\Delta k v_F \tau_c \ll 1$), we obtain for the shot noise (at $T = 0$):

$$S - S_{\rm cl} \approx \frac{e^2 v_F}{2\pi}\Delta k \langle T_1\rangle_\varphi (1 - \langle T_1\rangle_\varphi) \,. \quad (63)$$

Here we assumed $\delta x = 0$ as above. This formula follows by evaluating $S_{\rm fast} + S_{\rm fluct}$ in the limit of small Δk and using the sum rule (56). In contrast to the evaluation of $S_{\rm fluct}$ in the transport situation considered above, the integral over k' now runs over all states and is not restricted to a small transport window, which is essential to obtain (63). We conclude that this is yet another example[31] of a situation where the correct result for the shot noise cannot be obtained by taking into account only the "surplus" electrons in the transport window of size eV, even though this approach *does* yield the correct current. The presence of the filled Fermi seas is not merely important for the Nyquist noise contribution but for the shot noise as well. In the other limiting case, $\Delta k v_F \tau_c \gg 1$, we obtain the result expected from $S_{\rm slow}$, i.e. with $\langle T_1(1-T_1)\rangle_\varphi$ in Eq. (63), in addition to the "classical" contribution $S_{\rm cl}$ with its quadratic dependence on Δk.

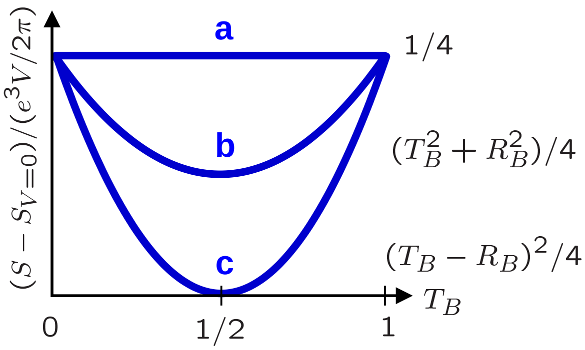

Figure 8: Shot noise as a function of the transmission of the second beamsplitter, for the fully incoherent case (and $T_A = 1/2$): Different models and parameter regimes lead to different curves (see text).

VIII. COMPARISON OF DIFFERENT MODELS AND REGIMES

In this section, we will collect and compare the various results obtained for the different models (and different regimes). We will restrict ourselves to the fully incoherent limit ($z = 0$), at $T = 0$ and $T_A = 1/2$. We emphasize, of course, that by comparing these models we do not want to imply that one should expect them to agree in any limit. The phenomenological classical model is simply a heuristic construction that is known to give the correct result for a single barrier, and even for the dephasing terminal ansatz it is not completely clear to which microscopic model it is to correspond. In addition, we remind the reader that the results obtained for dephasing by classical noise are not expected to coincide with those obtained for a more elaborate analysis of a quantum-mechanical bath, in the limit $eV \gg T$ considered here.

We have to distinguish three possible results for the modified partition noise term, entering the *shot-noise* $S - S_{V=0}$ (depicted in Fig. 8):

$$\begin{aligned}
&(a)\ \langle T_1\rangle_\varphi (1 - \langle T_1\rangle_\varphi) = 1/4, \\
&(b)\ \langle T_1(1-T_1)\rangle_\varphi = (T_B^2 + R_B^2)/4, \\
&(c)\ \left|\langle t_1^* t_2\rangle_\varphi\right|^2 = (T_B - R_B)^2/4 \qquad (64)
\end{aligned}$$

The corresponding values for the different models and regimes are indicated in the following table. Note that these expressions only refer to the contribution to S which is linear in voltage. For the model of dephasing by classical noise (last three entries), one still has to add

the constant background $S_{V=0}$, as well as S_{cl} (growing quadratically with voltage).

Model/regime	$\delta x \ll v_F/eV$	$\delta x \gg v_F/eV$
no dephasing ($z = 1$)	$T_1(\phi)(1 - T_1(\phi))$	b
Simple classical model	a	b
Dephasing terminal	b	b
"fast" environment	c	c
"slow" environment	b	b
"narrow electron beam"	a	a

For the particular parameters considered here, the presence of thermal averaging ($\delta x \gg v_F/eV$) only affects the results obtained without dephasing or from the simple classical model of Section IV. In any case, the results for the "slow classical noise" (Eq. (47)) and the dephasing terminal (Eq. (16)) both coincide with the result (b) $\langle T_1(1 - T_1)\rangle_\varphi$ obtained for complete thermal averaging (which is also obtained from the simple classical model if thermal averaging is present on top of dephasing). It might still be possible to deduce the presence of dephasing in the case of classical noise, both from the presence $S_{V=0}$ and S_{cl}, although we have to note that S_{cl} vanishes if both dephasing and thermal averaging are effective. The form of the shot noise S_{fast} (c) obtained in the limit of a "fast" environment (Eq. (44)) is not found in any of the other models. Finally, the result (a) $\langle T_1\rangle_\varphi (1 - \langle T_1\rangle_\varphi)$ conjectured from the simple classical model (in the absence of thermal averaging) can also be found for dephasing by classical noise, provided we consider a special transport situation, with a "narrow beam of electrons" (Eq. (63)).

IX. CONCLUSIONS

We have analyzed the effect of a fluctuating environment on the shot noise in an electronic Mach-Zehnder interferometer. The environment has been modeled as a classical noise field which leads to a fluctuating phase difference for electrons traversing the interferometer and thereby suppresses the interference term. For comparison, we have also discussed a simple classial ansatz and the phenomenological dephasing terminal approach.

The effect of dephasing on the average *current* is always the same, and qualitatively indistinguishable from "thermal averaging" (averaging over wave number in the presence of a path-length difference). However, important differences appear in the shot noise results. While the power spectrum of the phase fluctuations does not enter the current for the case of a classical fluctuating potential considered here, the current *noise* strongly depends on the fluctuation spectrum, thus offering more information on the environment. There are three main contributions to the current noise: some "classical" current noise (rising like V^2), due to the fluctuations of the conductance, some "Nyquist noise" background, and finally the usual partition noise, modified due to the presence of the environment. The partition noise contribution depends on a two-time correlator of four transmission amplitudes and is sensitive to the power spectrum. We have distinguished the limits of a "slow" and a "fast" environment, depending on whether the inverse correlation time of fluctuations $1/\tau_c$ is much smaller or much larger than the maximum of voltage eV and temperature T. We have found that the usual result $T_1(1 - T_1)$ for the partition noise (at given transmission probability T_1) may be replaced by one of three limiting forms, depending on the correlation time τ_c, the transport situation and the dephasing model: (i) For a "slow" environment, the usual result is averaged over the phase fluctuations, $\langle T_1(1 - T_1)\rangle_\varphi$, which is similar to the effect of thermal averaging and identical to the result provided by the dephasing terminal (although there may be problems with the dephasing terminal, see Section V). (ii) For a "fast" environment applied to a nearly mono-energetic beam of electrons, we obtain $\langle T_1\rangle_\varphi (1 - \langle T_1\rangle_\varphi)$, which is also the result derived from a simple classical model. (iii) For a "fast" environment applied to the usual transport situation (with the chemical potential of one of the input reservoirs increased by eV), we obtain $\left|\langle t_1^* t_2\rangle_\varphi\right|^2$, where $t_{1,2}$ are the amplitudes of reaching the output port from inputs 1 and 2 ($|t_1|^2 = 1 - |t_2|^2 = T_1$). In this case, the shot noise at $T = 0$ can even be suppressed to zero by the fluctuating environment for appropriate parameter combinations, while on the other hand the Nyquist noise becomes nonzero.

We have discussed the crossover between "slow" and "fast" environment, the dependence of the shot noise on the phase difference between the paths and on the beam splitter transparency, and the influence of finite temperatures and finite path-length difference (thermal averaging).

The most important tasks that remains to be tackled in future works are the consideration of finite frequency shot noise, the derivation of realistic microscopic power spectra as input for this calculation, and, in particular, the inclusion of a truly quantum-mechanical environment, which will be relevant particularly for the case of voltages larger than temperature.

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